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ABSTRACT

This paper is concerned with the role of spreadsheets as a tool for the development of mathematical models in science, one aspect of a collaborative project which worked with two groups of pre-university students from Mexico and the United Kingdom. The purpose of the modeling activities designed was to engage students in creating an "artificial world" as a window into a science phenomenon to be explored and studied in detail. The models proposed were a combination of "exploratory" and "expressive" activities. Students constructed spreadsheet models guided by paper-based worksheets. They were asked to construct graphs relating variables of the model and to investigate the effects of varying different parameters, corresponding to different physical situations. It is concluded that the spreadsheet provides the opportunity to connect different mathematical representations and retains aspects of the science problem within the spreadsheet layout, functions, and the representations themselves. (ASK)

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Mathematical Practices in the Sciences: The Potential of Computers as a Modelling Tool

by
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Mathematical Practices in the Sciences: The Potential of Computers as a Modelling Tool

Susan Molyneux-Hodgson and Simon Mochon

Introduction

This paper is concerned with the role of spreadsheets as a tool for the development of mathematical models in science, one aspect of a collaborative Mexico/UK project. We worked with two groups of pre-university students, one group in Mexico City and one group in London, UK.

A mathematical (algebraic) approach to modelling of science separates out the physical context, placing the emphasis on the mathematical objects and transformations needed to solve a problem. However, mathematical modelling requires movement between the physical situation being modelled and the mathematical representations of that model (Mason & Davis, 1991). We introduced the spreadsheet as an innovation into the students' science classroom to investigate whether the computer environment could support students in retaining links between different mathematical representations and the science being modelled. The structure of, and representations within, the spreadsheet retain aspects of the scientific problem, for example the labelling of columns and graphs, which potentially support students in maintaining links. In the manner of Teasley and Roschelle (1993 p. 229) the spreadsheet was "intended to both enable and mediate student's learning".

The purpose of the modelling activities we designed was to engage students in creating an "artificial world" (Mellar et. al., 1994) as a window onto a science phenomenon, to be explored and studied in detail. The models we proposed were a combination of "exploratory" and "expressive" activities (Mellar et al, 1994 p. 3). Exploratory models are ones in which learners explore models provided by someone else, for example a teacher, and expressive models are ones in which learners can express their own ideas. The students we worked with constructed spreadsheet models guided by paper-based worksheets. They were asked to construct graphs relating variables of the model and to investigate the effects of varying different parameters, corresponding to different physical situations.

A spreadsheet can give access to essentially three types of mathematical representation, the table, formula and graph. It is these representations which we initially conjectured could be accessed as resources for problem-solving by the students. Movement between these representations is overt in the spreadsheet environment and thus may support students in constructing an image of the relations between physical objects from several perspectives. We contend that there are also structural and functional aspects of a spreadsheet which make links to the physical more transparent. For example, the function **Define Name** enables individual spreadsheet cells to be given a label, making the link between a spreadsheet formula and a physical object more concrete. We encouraged this practice by

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sometimes specifically asking students to **Define Name**(s) for cells which would contain constants or parameters (e.g. 'gravitational constant' in Artificial Satellites and 'step size' in Oscillations & waves). Thus, when entering formulae, they could include labels for constants and parameters, maintaining an overt connection to the physical situation. In addition, the tabular layout of the spreadsheet means that students' work is available to them visually at all times which has implications for the ease of de-bugging processes.

An important aspect of our work is the idea that it is *person(s) acting with mediational means* which is a unit in the analysis (Wertsch, 1991). This means that we do not attempt to separate students' mental action from the tools they are using. From the point of view of this study, the spreadsheet environment, and more specifically the spreadsheet external representations, are potential mediators of modelling activity. We conjectured that through activity with the spreadsheet students would make links between their informal problem-solving approaches and more formal representations of mathematical models.

Mediational means are sociocultural and thus taking the wider 'setting of activity' into account is necessary, as we were working in two countries. In this respect we were influenced by the work of Lave (1988) which suggests that mathematics can surface in different forms within different settings. The idea that mathematics can give structure to, or be structured by, other ongoing activities also seems important with regard to students carrying out mathematical modelling activities within different mathematics and science cultures.

With these ideas in mind, we have probed how students participate in mathematical modelling activities within science and whether the computer as a tool is used as a resource by students. Questions we ask thus include, what resources are drawn upon to structure activity ? and what role does a spreadsheet play in this activity ?

Methods and data collected

Students worked on several modelling activities over a period of six months during one school year. Students in both countries developed mathematical models within physics, chemistry and biology in topic areas such as diffusion, population growth, chemical equilibrium and artificial satellites. The modelling activities were designed to fit in, as far as possible, with ongoing topics in the respective science curricula. It was not possible to match science topics in both countries for the whole of the study and so students worked on some modelling activities which were not in common (see Table 1). They worked on the spreadsheet models during appropriate science lessons, in the usual science classroom with their normal teacher often present as well as one or two researchers.

The science situations to be modelled were presented to the students on paper. A worksheet was used to combine some subject information with sections for students to record their ideas, give

explanations, predict graphs and consider limitations. This approach was aimed at ensuring that the students took control of the modelling process immediately and were not reliant on a lengthy exposition by the teacher or researcher. The worksheets were produced collaboratively by researchers in the two countries and took extensive time to refine.

The worksheet usually required students to carry out some numerical calculations related to the science situation as a first step. The intention was that the student could draw on these numerical calculations in developing their spreadsheet model. In the cases where a mathematical model pre-existed (e.g. artificial satellites, for which students were presented with the formula for circular motion) qualitative questions relating to the science situation were posed. In both cases, the idea was to provide an engaging lead into the problem and to support the students in constructing the mathematical model being proposed.

Individual interviews were conducted, with students either presented with models they had constructed in class, or students being asked to work on a previously unseen modelling problem. All forms of available data were analysed including: field notes and video recordings of classroom-based modelling sessions; video recordings of individual interviews with case study students; and records of students' paper-based and computer-based work.

To match the topics of the science curricula in each country, the two groups worked on slightly different sets of modelling problems. The titles of the worksheets designed for the modelling activities are shown in Table 1.

Biology	Chemistry	Physics
Diffusion	Chemical Equilibrium (Mex.)	**Inelastic Collisions**
Population Growth	Environmental Pollution (Mex.)	**Elastic Collisions**
Population Genetics (UK)	Periodicity (UK)	**Gravitation**
	Lattice Energies (UK)	**Artificial Satellites**
		Oscillations & waves (UK)

Table 1

(titles in bold type were presented in both countries)

Neither of the student groups had experienced "mathematical modelling" in their science studies in an explicit way, prior to the research, although a sub-set of the UK students (who were studying post-compulsory mathematics) were being introduced to modelling in mathematics. In both Mexico and the UK, before the commencement of the spreadsheet activity, most students had difficulty articulating their understanding of a mathematical model and several professed to have never heard of the idea.

Short spreadsheet training courses were delivered in each country, prior to commencement of the study, where students worked on science and mathematics problems in order to learn spreadsheet functions.

This paper is divided into three sections as a means of enabling us to discuss aspects of students' modelling work. It should be noted however, that although we have separated out ideas to talk about them, the analysis could have been "cut" in a different manner. The ideas we will discuss are i) students' interaction with a spreadsheet, as a mediator between the science modelled and mathematical representations of that model; ii) students' use of spreadsheets as a resource within problem solving (a resource which structures students' thinking); iii) cultural influences on students' use of spreadsheets.

Students' acting with spreadsheet as mediational means

Retaining links between the science situation being modelled and the mathematical representations of the model is an important aspect of successful modelling activity, especially for these science students where the emphasis was not so much on abstracting from the physical situation in order to manipulate mathematically before returning to the physical. Here, the aim was to support students in making sense of the science situation by making the spreadsheet available as a resource. By focussing on the science <-> model links at various stages in the modelling process, a student can continuously check whether the model makes sense from a science viewpoint and also check that the model represents the science situation appropriately.

The following example illustrates the ways in which the spreadsheet environment offers mechanisms which make it more likely that students do not lose sight of the fact they are modelling a physical phenomenon. We found that if students could not make sense of the feedback from the spreadsheet table and/or graph with respect to their formal or everyday knowledge of the science situation, they then questioned the formal spreadsheet model which they themselves had constructed. For example Mae, after inputting a formula to work out the distance - time characteristics of a car, plotted the graph but thought it looked incorrect.

S1 I expected it to increase at first, before doing anything, but it went straight into negative numbers. They were quite big negative numbers and didn't look very realistic.

Mae had in fact used an incorrect formula to produce the graph. The instant feedback provided by the graphical representation allowed her to question the physical situation again. She related her spreadsheet model to her prior expectation of what the representation of car motion would look like. In this way, the interaction of her expectations and the spreadsheet feedback led her to re-evaluate her approach. The fact that students always entered the spreadsheet formulae themselves may be critical here. They may be less likely to question ready-made expressions such as the way models are presented within some learning environments (Beare, 1993). We maintain that the students' construction of the spreadsheet model provoked movement between, and a re-questioning of, the formal <-> physical relationship.

In a similar way, another UK student, Betty, interacted with feedback from her spreadsheet tables to iteratively change a model of radioactivity decay, set in a biological scenario of cancer therapy. An

algebraic expression for decay was presented on paper to support the students in their model construction as this model was known to be unfamiliar to the non-physics students, of which Betty was one. The expression given on paper was,

$N = N_0 e^{-\lambda t}$, (where N = amount of substance; N_0 = initial amount; λ = decay constant; t = time).

After entering an incorrect rule in the spreadsheet (the equivalent of "previous amount times decay constant") Betty realised from the table produced that the amount of cobalt in the body was decreasing very rapidly. The formula Betty used was effectively "Nλ" which in her spreadsheet language was C3*cobalt60. After noticing that the amount of cobalt reached zero after only 7 years, she rounded the numbers first to two decimal places and then to four decimal places. She then modified her formula by dividing by 100, as shown in Figure 1.

	A	B	C	D
1		No of year	Cobalt units present	
2				
3		0	100	
4	0.13	1	=(C3*cobalt60)/100	
5		2		

cell Named "cobalt60"

Figure 1. Betty's second formula for decay

She was modifying her formula, in the light of feedback from the tabulated numbers, in an attempt to produce data which made sense to her. What she considered to be 'sensible' data is not known but may be related to prior knowledge concerning the application of radioactivity to cancer treatment. She seemed unaware that the algebraic formula represented a theoretical model which constrained the problem and instead she used a 'trial and refinement' approach to modify her spreadsheet formula. The 'trial and refinement' approach is explicitly taught to UK students in their pre-16 mathematics syllabus. Betty's work was also influenced by her expectations of what a school question would ask for, for example, at a later stage she entered another incorrect rule. She stated she knew it was not correct because 'zero' (of amount of cobalt) was reached before 30 years and the question would not have asked for calculations up to 30 years if this were the case. Although Betty is making links between the science and the mathematics here she still inappropriately changes the formulae and a teacher needed to intervene to support her, the teacher being a resource she could call on.

It was mentioned previously that use of the function **Define Name** is a potential resource to support links between the science situation and the mathematical model. Although the use of **Names** was explicitly promoted on our worksheets, students did not always choose to use this approach. The use of **Names** did however foster a discourse for solving problems encountered in the activity, e.g. in mediating communication between student and teacher, as in the case of Jonah. In his spreadsheet work Jonah was adamant about using **Names**. This may be related to his need to keep contact with the physical objects of the problem which was observed in the 'normal' science classroom as well as the spreadsheet work. Jonah, another student and a researcher were considering Jonah's spreadsheet together, trying to sort out a problem with it,

Jonah Why don't I get anything ?
RS You've got minus 'gc' which is the ... gravitational force
Jonah Gravitational constant
RS Times the ...
Jonah Mass of earth times the mass of satellite, ... all divided by radius of orbit squared.

Jonah used physical object names in his talk, even though he had **Named** those objects 'gc', 'moe', 'mos' in the spreadsheet. The other student present had difficulty engaging in this discussion, perhaps because she was one of several students who used the dollar sign (e.g. A2) convention for absolute referencing rather than **Define Name**. The use of **Names** supported Jonah and the tutor towards resolving a major problem with Jonah's model.

The issues of *feedback* and *spreadsheet structure* as they relate to supporting students to make links between the science and mathematics were also evident within the activities of the Mexican students. In Mexico the use of the spreadsheet structure to mediate between the physical and mathematical was explicit in the case of a model for diffusion, presented as the diffusion of molecules through imaginary compartments within a living cell. In this model, each column of the spreadsheet represented one compartment of the living cell. As students made sense of their models, and analysed their data, they moved their fingers across the columns of the spreadsheet, referring at the same time to the movement of molecules between compartments.

The graphical representation provided powerful feedback for the Mexican students as illustrated by the case of an elastic collisions' model. Students were asked to predict the graphs they expected from plotting velocity as the mass of a moving object varied. Most of them drew a decreasing function (which is correct) but they did not consider that their graph might cross into a region of negative velocities. When they plotted the graph on the spreadsheet, they noticed that values for velocity became negative after a certain value for the mass. They questioned this result and after a time reached an interpretation of the graph. One student said, "it (the ball) will move backwards", demonstrating a link between the mathematical model and the actual physical situation.

Students' use of the spreadsheet as a resource

As well as supporting students to move between the mathematical model and the science, the spreadsheet clearly structured the ways in which students worked. In cases where there was a choice of tools to solve a problem students would choose to use the spreadsheet, often for explicitly practical reasons, e.g. the Mexican students who said "it was faster". However, as well as issues of speed, other students began to understand that the spreadsheet could be used more as a tool for thinking with, for example the UK student who said, "I was just going to do it manually myself, but I decided it would be easier to use a spreadsheet ... I was trying to think of a formula (by himself) and I found it too hard ... and that's just what the spreadsheet's doing really". In this situation Alex used the spreadsheet to support his construction of a general formula, which he had found difficult to do from his specific calculations on paper.

The most common scenario when students worked on modelling problems was that they *moved* between the use of resources, for example, the spreadsheet, paper and calculators, in an interactive way. One example of this was a UK student, Alex, when investigating car motion described by a given model, $x = at^2 - t^3$ (where x = position of car; a = a constant; t = time). When Alex was interviewed immediately following this problem, he said that he used his calculator to validate an answer estimated from a spreadsheet graph. The question posed was 'At what time will the car pass position x = 75 ?'

I3. 18. it's easier to use the graph to read off ... and then I worked it out on my calculator as well, with the time that I estimated, to see whether it was roughly right.

He had estimated t = 3.7 from the graph of $x = at^2-t^3$. He substituted this estimate into the equation to see what value of x was obtained. He used this process to alter his initial estimate, ending up with the value t = 3.8. There was a seamless movement between resources, structuring his overall, ongoing activity. He did not work solely with the equation but moved between the graph and the equation using a kind of iterative approach. The UK students were initially not confident in the use of algebraic equations as formal algebra is not a significant part of their pre-16 experience and Alex's case demonstrates that work with the spreadsheet did support students in making sense of the different mathematical representations.

In the construction of a spreadsheet model, the variables are usually presented in columns and their relationships with each other have to be made explicit with spreadsheet formulae. Analysis of videos of students constructing spreadsheet models indicates that the spreadsheet acted as a structuring resource in identifying variables within a model. Two UK students working on the radioactive decay model mentioned earlier (where the expression $N = N_0e^{-\lambda t}$ was provided on paper). As they began to construct the model on their spreadsheet they pointed at columns and cells, using both gesture and the mouse, and related these to various elements in the formula.

> "This column is going to be (pointing to the column) the amount of substance ... at a time. So N is the amount that we come out at the end with ... so it's *that*, which is N_0, which is the 100 ... times 0.13."

Students also used the function **Define Name** to separate absolute from relative spreadsheet references, supporting them to differentiate between the parameters and variables within the formal model.

Students' prior experience was found to influence the ways in which they approached the spreadsheet work. This is clearly illustrated in the example of Nisha's use of the spreadsheet table as a resource to structure her problem solutions, drawing on a previously taught 'trial and refinement' algorithm to shape her spreadsheet use. Whilst modelling car motion, she moved around the spreadsheet producing tables of different step size to answer different problems and also to do specific calculations. She described her use of the table thus,

Nisha So from the above table I knew that 75 is between 2 and 2.5 so 75 is between 2 and 3 and 2.5 was 84 so it is between 2 and 2.5 and then I did first 2.1 and went up in .1 but that was not accurate enough so I did 2.01.

The spreadsheet, and her 'trial and refinement' strategy, together structured her activity. However the example illustrates that the 'trial and refinement' strategy was a more dominant resource as there were more efficient ways of solving this problem using the spreadsheet, for example by use of **Define Name**. She would carry out trial and refinement procedures quite often, always using a calculator.

Cultural influences on students' use of spreadsheets

As mentioned in the previous section students' previous experiences, including the school science and mathematics cultures, clearly structured the ways in which the students worked. The paper on "Cultural Influences on Classroom Participation" (Rojano & Sutherland, 1997) described some aspects of the school cultures the students were situated within. For example, how the Mexican students generally liked to be precise and work with algebraic representations and the UK students were confident with graphs but not with algebra.

Differences in the students' use of spreadsheet formulae was one aspect which could be traced to previous school mathematics experience. For example during the collisions model, expressions used by the UK students for momentum were of two kinds, basically

$$= (m_1 * v_1) + (m_2 * v_2)$$
$$= (m_1 * v_1) + (m_2 * -v_2)$$

The first expression here was used by students who had used -6 in their 'velocity of person moving to left' column, i.e. direction was accounted for elsewhere. In both cases the students associate the negative sign with the magnitude of the velocity, keeping close to the physical idea.
They also used brackets to group objects here. It is possible that students are thinking about the physical object 'momentum' and therefore group the elements 'mass' and 'velocity' together. This use of brackets is certainly not necessary from an arithmetic (i.e. order of operations) viewpoint.
This contrasts with the students in Mexico where the evidence indicated that mathematics was more structuring, than being structured by, the physical situation. The majority of the Mexican students used a spreadsheet formula representing the relation,

$m_1v_1 - m_2v_2$ where v_1 and v_2 are both positive.

Whilst all the above-quoted formulae model the collisions situation appropriately, we suggest that the UK students remained closer to the physical than the Mexican students and that this can be related to their school culture experience (as described in "Cultural Influences on Classroom Participation" (Rojano & Sutherland, 1997)).

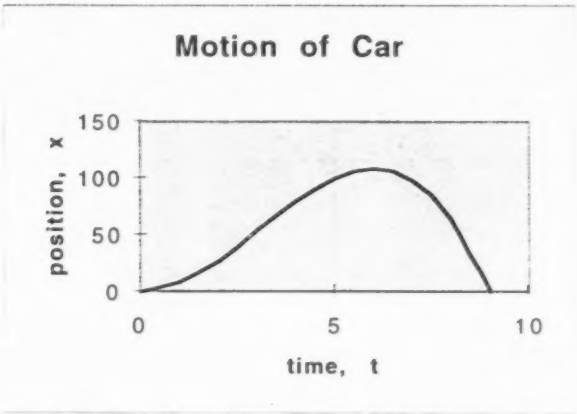
From classroom observations before the spreadsheet was introduced, it was evident that for the UK students the calculator was one resource consistently drawn upon. The difference in amount of use of calculators as a tool between the countries was marked. Accordingly, as described above, the calculator was observed to structure UK students' spreadsheet activity in a way which was not

apparent in Mexico. For example, Nisha constructed her spreadsheet formulae a bit-at-a-time, in the same way as you might keep pressing <enter> on a calculator to give a running answer. An example of this was the series of formulae she entered with the purpose of entering a formula for speed (instantaneous speed in circular motion). She entered the following sequence of spreadsheet formulae, pressing <enter> each time,

= (2 π()	<enter>
= (2*π()*A8/)	<enter>
= (2*π()*A8/A5)	<enter>

Again, this exemplifies the interplay between the spreadsheet and previous knowledge with calculators, mutually constituting activity. However, this situation did not arise in Mexico as the students were much less likely to draw upon calculators as resource in the modelling activities. Mexican students did make use of calculators during spreadsheet work but it was far less extensive.

The differences in approaches to a problem, experienced by students from the different countries, is demonstrated by the example of two students - Maria (a Mexican student) and Alex (a UK student), modelling car motion. Given the expression $x = at^2-t^3$, where x = distance, t = time and 'a' was a specified constant, both students successfully produced a table of values from the equation and plotted a distance - time graph, which shows the motion reaching a maximum before decreasing.



Maria initially started to use an inappropriate formal rule (v = d/t) to give the velocity of the car at t=6. The interviewer intervened to ask whether she thought that using this expression was appropriate and then she realised that there was a maximum at this point in the graph and therefore she concluded that the velocity must be zero by applying a physical argument, "it is zero, because it (the car) goes backwards". This suggests that formal mathematics is a strong structuring resource for Maria when scientific problem-solving is the ongoing activity. Alex immediately correctly interpreted the meaning of the maximum in the speed-time graph and did not attempt to use a more algebraic approach, the graphical representation being used to structure his responses.

The example of Maria and Alex above exemplifies one aspect of the cultural influences on students' modelling activity with spreadsheets. That Maria accesses an equation and Alex a graph to answer the 'same' question, is not atypical and examples of the 'need for exactness' of Mexican students and the graphical preferences of UK students were manifest throughout the study, in modelling and non-modelling sessions (Sutherland et. al., 1996). Thus, initially, student's activity was shaped by their use of representations which mirrored their use in paper-based work.

However, over the period of the study, a shift in students' preferences was observed i.e. the repertoire of resources they accessed increased and we take this as evidence that they were taking the spreadsheet on board as a tool for thinking with. The UK students became more confident in the use of algebraic representations, including those students who had not continued with their mathematics studies. Having access to algebraic information in a visual form seemed a positive aspect of spreadsheet use to the UK students,

Ella ... (with) the computer you can actually see what you are typing in. With the calculator you do 2 + 2 and let us say you want to add 2 three times, so 2 + 2 and then it becomes 4 and then + two, it doesn't really display what's going on ... its important (the spreadsheet), I mean its written out here, the calculator will just give me 18 but here it tells me C5 x Lambda = 18.

We conjecture that work with the formal representation of equations in the spreadsheet environment, and their links to the graphical representation, supported the UK students in understanding algebraic representations on paper.

Summary

The spreadsheet seems to offer students a new psychological tool (Wertsch, 1991) for developing mathematical models in science. The spreadsheet provides the opportunity to connect different mathematical representations and retains aspects of the science problem within the spreadsheet layout, functions and the representations themselves.

The computer feedback from the formal spreadsheet model often provoked students to re-examine their assumptions about the physical situation thus forcing them to make connections between the mathematical model and the science being represented. In this way the spreadsheet mediated the necessary linking between scientific and mathematical aspects of the problem.

Students made sense of their spreadsheet models from the physical viewpoint, sometimes with respect to more formal physics knowledge, *"the [centripetal] force is lower 'cos it's [the satellite] higher, 'cos it's escaping from the field"* and sometimes with respect to more everyday knowledge *"I think if we don't have zero years, it would work, 'cos there's no such things as zero years ... cos you can't inject someone in zero years can you!"*. We conjecture that the dynamic nature of the spreadsheet

feedback provokes students to make links between the physical situation being described and the formal models, which similar work with paper cannot provide.

The sense which students made of the model and the ways in which they worked with the spreadsheets was influenced by previous school experience, particularly the school mathematics culture. As students engaged in modelling activity with the spreadsheet, the activity was structured by multiple instances of resources being brought to bear. The resources students drew in were also related to previous experience and so differed between the two countries.

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